Item	Description
	Measurement of reflection coefficient magnitude of reflector at a single frequency of the following waveguide sizes terminated with standard waveguide connectors.
201.920a-1	WR90 (8.2–12.4 GHz)
201.920a-2	WR62 (12.4–18.0 GHz)
201.920z	Special calibrations not covered by the above schedule.

HIGH-FREQUENCY REGION

201.830 Immittance.

1) Maximum accuracy can be achieved only in the case of instruments and components equipped with connectors having a plane of reference directly compatible with the Bureau system with no necessity for special adapters. To preserve higher calibration accuracies, coaxial connectors should be utilized on standard instruments and components wherever possible.

2) Power applied to any component under test normally will not exceed 1 w. In this respect, if necessary, caution should be clearly stated in the calibration request.

Item	Description
201.830a-1	Two-terminal impedance measurement at one point in the frequency range 30 kHz to 400 kHz, 0 to 10,000 ohms resistance, and 0 to 1100 μh inductance.
201.830a-2	Each additional point within the limits in item 201.830a-1.
201.830b-1	Two-terminal impedance measurement at one point in the frequency range 30 kHz to 1 MHZ, 0 to 1000 ohms resistance, and 0 to 110 μh inductance.
201.830b-2	Each additional point within the limits in item 201.830b-1.
201.830c-1	Two-terminal admittance measurement at one point in the frequency range 30 kHz to 1 MHz, 0 to 1100 pf capacitance.
201.830c-2	Each additional point within the limits in item 201.830c-1.
201.830d-1	Two-terminal admittance measurement at one point in the frequency range 5 MHz to 250 MHz, 0 to 50 μmho conductance, and 0 to 50 pf capacitance.
201.830d-2	Each additional point within the limits in item 201.830d-1.
201.830e-1	Two-terminal impedance measurement of coaxial components at frequencies from 50 MHz to 1 GHz, within the ranges 0.5 to 5000 ohms magnitude and 0 to 90° for phase angle.
201.830e-2	Each additional point within the limits in item 201.830e-1.
201.830f-1	Q-Standard calibration in the frequency range 50 kHz to 45 MHz, 0 to 1000 for effective Q, and 30 to 450 pf effective resonating capacitance.
201.830z	Special two-terminal immittance calibrations not covered by the above schedule.
201.831a-1	Three-terminal capacitance calibration at 100 kHz, 465 kHz, or 1 MHz for the following fixed nominal values: 10^{-2}, 10^{-1}, 10^{0}, 10^{1}, 10^{2}, and 10^{3} pf, per frequency.
201.831b-1	Three terminal capacitance calibration at 465 kHz at one point in the range 0.001 to 100 pf.
201.831b-2	Each additional point within the limits in item 201.831b-1.

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Equivalence of Different Integral Equations for Confocal Resonators

Lotsch[1] has proposed an integral equation for the modes of a two-dimensional confocal resonator, which he says is different from and more accurate than the integral equation used by earlier workers, such as Boyd and Gordon.[2] Actually the two integral equations are completely equivalent. Lotsch's equation describes the field at the midplane of the resonator, and the simpler equation of Boyd and Gordon describes the field at either of the confocal mirrors.

To see the equivalence, denote the field at the mirror by $u(x)$ and the field at the midplane by $X(x)$. The two fields are related by the Fresnel diffraction formula,

$$X(x_0) = \frac{e^{i(\pi/4-kd/2)}}{\sqrt{\lambda d/2}} \int_{-a}^{a} u(x_1) \exp\left\{\frac{ik}{2d}\left[x_1^2 - 2(x_0 - x_1)^2\right]\right\} dx_1, \tag{1}$$

$$u(x_1) = \frac{e^{i(\pi/4-kd/2)}}{\sqrt{\lambda d/2}} \int_{-\infty}^{\infty} X(x_0) \exp\left\{\frac{ik}{2d}\left[x_1^2 - 2(x_0 - x_1)^2\right]\right\} dx_0, \tag{2}$$

where $2a$ is the width of the confocal mirrors, d is their total separation, and $k=2\pi/\lambda$, where λ is the wavelength.

The integral equation for $u(x)$ is obtained by eliminating $X(x)$ from (1) and (2). We get

$$\begin{aligned}\kappa u(x_1) &= \frac{2e^{i(\pi/2-kd)}}{\lambda d}\left[\int_{-\infty}^{\infty} dx_0 \int_{-a}^{a} dx_2 u(x_2) \exp\left\{\frac{ik}{2d}\left[x_1^2 + x_2^2 - 2(x_0 - x_2)^2 - 2(x_0 - x_1)^2\right]\right\}\right] \\ &= \frac{e^{i(\pi/4-kd)}}{\sqrt{\lambda d}} \int_{-a}^{a} u(x_2) e^{ikx_1x_2/d} dx_2,\end{aligned} \tag{3}$$

after carrying out the integration over x_0. Eq. (3) is just the integral equation of Boyd and Gordon, and its solution in terms of prolate spheroidal wave functions is well known.

On the other hand, if we eliminate $u(x)$ from (1) and (2), we get

$$\begin{aligned}\kappa X(x_2) &= \frac{2e^{i(\pi/2-kd)}}{\lambda d}\left[\int_{-a}^{a} dx_1 \int_{-\infty}^{\infty} dx_0 X(x_0) \exp\left\{\frac{ik}{2d}\left[x_1^2 - 2(x_2 - x_1)^2 + x_1^2 - 2(x_0 - x_1)^2\right]\right\}\right] \\ &= \int_{-\infty}^{\infty} K(x_2, x_0) X(x_0) dx_0,\end{aligned} \tag{4}$$

where

$$K(x_2, x_0) = \frac{2e^{i(\pi/2-kd)}}{\lambda d} \exp\left(\frac{2ikx_0x_2}{d}\right) \int_{-a}^{a} \exp\left[-\frac{ik}{d}(x_1 - x_0 - x_2)^2\right] dx_1. \tag{5}$$

Up to unimportant differences in notation and misprints, (4) is Lotsch's integral equation.

It follows that Lotsch's equation is just Boyd and Gordon's equation written in a more complicated form. If correctly handled, therefore, the two equations must lead to identical physical predictions.

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[1] H. Lotsch, "The confocal resonator system with a large Fresnel number (V-type eigenmodes)," IEEE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-12, pp. 482–483; July, 1964.

[2] G. D. Boyd and J. P. Gordon, "Confocal multimode resonator for millimeter through optical wavelength masers," *Bell Sys. Tech. J.*, vol. 40, pp. 489–508; March, 1961.